Strength of materials

References

(1) "**Strength of Materials**", Ferdinand L. Singer , 3rd Edition, 1980.
(2) **Introduction to mechanics of Materials** " , Paul E. Popov, 2nd Edition" 1956.
(3)**Elements Strength of Materials**", Stephen P. Timoshenko and D. H. Young, 5th Edition , 2011.

Strength of materials:

Strength of materials extends the study of forces that has begun in engineering mechanics the difference between engineering mechanics and strength of materials is that engineering mechanics cover relations between forces acting on rigid bodies, while strength of materials deals with the relations between externally applied loads and their internal effects on bodies. Also the bodies are no longer assumed to be ideally rigid. The deformations, however small are of major interest.

Simple Stresses

Simple stresses are expressed as the ratio of the applied force divided by the resisting area.

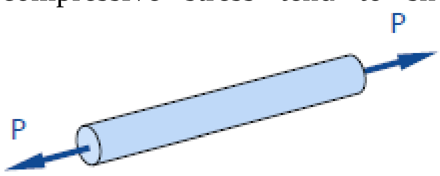
Stress = Force / Area

It is measured by unit force over unit area
It is measured in English unit by psi or ksi or their units
and in SI unit by (kN/m^2) or (N/m^2=Pa) or (N/mm^2=MPa) or other units.

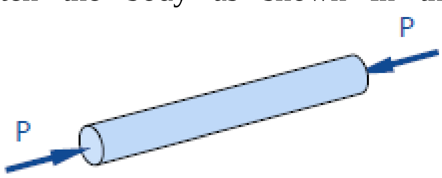
Simple stress can be classified as
1. Normal stress,
2. Shear stress
3. Bearing stress.

1. Normal stress

This type of stress develops when a force is applied perpendicular to the cross-sectional area of the material. There are two types of normal stresses; tensile stress and compressive stress. Tensile stress applied to bar tends the body to elongate while compressive stress tend to shorten the body as shown in the Fig. below.



Bar in Tension



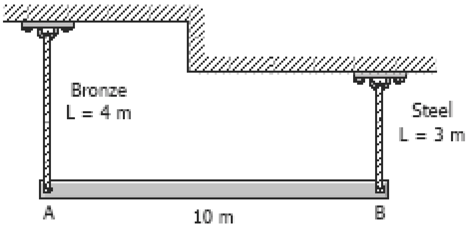
Bar in Compression

The magnitude of normal stress is calculated by the formula:

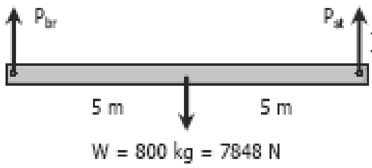
$$\sigma = \frac{P}{A}$$

where σ is the normal stress, P is the applied normal load and A is the area.

Ex.1: A homogeneous 800 kg bar AB is supported at either end by a cable as shown in Fig. below. Calculate the smallest area of each cable if the stress is not to exceed 90 MPa in bronze and 120 MPa in steel.



Solution:



By symmetry:

$P_{br} = P_{st} = \frac{1}{2}(7848)$

$= 3924$ N

For bronze cable:

$P_{br} = \sigma_{br} A_{br}$

$3924 = 90 A_{br}$

$A_{br} = \mathbf{43.6\ mm^2}$

For steel cable:

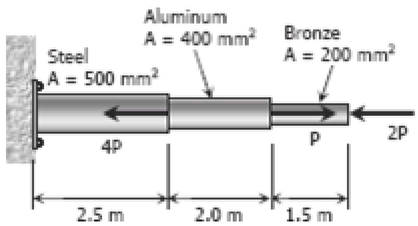
$P_{st} = \sigma_{st} A_{st}$

$3924 = 120 A_{st}$

$A_{st} = \mathbf{32.7\ mm^2}$

Ex.2: An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in Fig. below. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa.

Solution:



For bronze:

$\sigma_{br} A_{br} = 2P$

$100(200) = 2P$

$P = 10\ 000$ N

For aluminum:

$\sigma_{al} A_{al} = P$

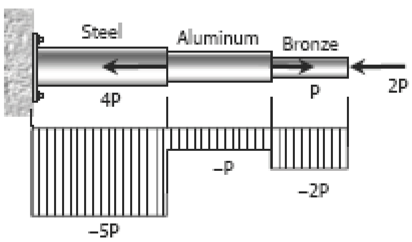
$90(400) = P$

$P = 36\ 000$ N

For Steel:

$\sigma_{st} A_{st} = 5P$

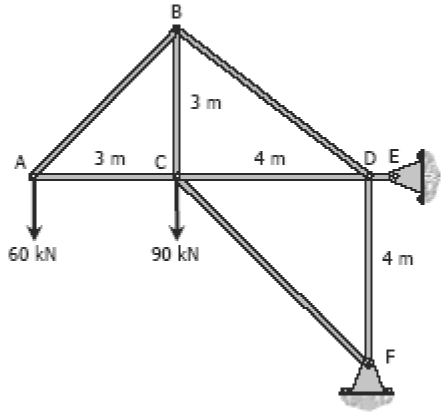
$P = 14\ 000$ N



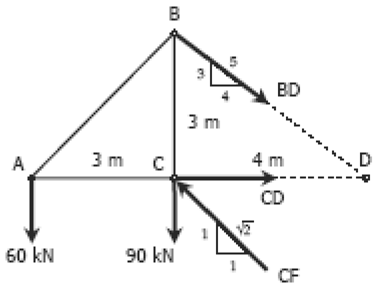
Negative (−) means compression

For safe P, use $P = 10\ 000$ N $=$ **10 kN**

Ex3: Find the stresses in members BC, BD, and CF for the truss shown in Fig. below. Indicate the tension or compression. The cross sectional area of each member is 1600 mm^2.



Solution:



FBD 01

For member *BD*: (See FBD 01)

$\Sigma M_C = 0$

$3(\frac{4}{5}BD) = 3(60)$

$BD = 75$ kN Tension

$BD = \sigma_{BD} A$

$75\ (1000) = \sigma_{BD}\ (1600)$

$\sigma_{BD} = 46.875$ MPa (Tension)

For member *CF*: (See FBD 01)

$\Sigma M_D = 0$

$4(\frac{1}{\sqrt{2}}CF) = 4(90) + 7(60)$

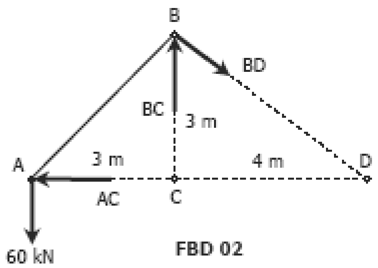
$CF = 195\sqrt{2}$

$\quad = 275.77$ kN Compression

$CF = \sigma_{CF} A$

$275.77\ (1000) = \sigma_{CF}\ (1600)$

$\sigma_{CF} = 172.357$ MPa (Compression)



FBD 02

For member *BC*: (See FBD 02)

$\Sigma M_D = 0$

$4BC = 7(60)$

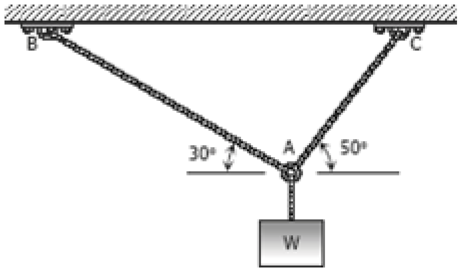
$BC = 105$ kN Compression

$BC = \sigma_{BC} A$

$105\ (1000) = \sigma_{BC}\ (1600)$

σ_{BC} **= 65.625 MPa (Compression)**

Ex.4.Determine the largest weight W that can be supported by two wires shown in Fig. below. The stress in either wire is not to exceed 30 ksi. The cross-sectional areas of wires AB and AC are 0.4 in^2 and 0.5 in^2, respectively.



Solution:

For wire *AB*:

By sine law (from the force polygon):

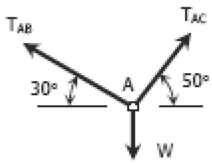
$$\frac{T_{AB}}{\sin 40°} = \frac{W}{\sin 80°}$$

$T_{AB} = 0.6527W$

$\sigma_{AB}A_{AB} = 0.6527W$

$30(0.4) = 0.6527W$

$W = 18.4$ kips



FBD of knot A

For wire *AC*:

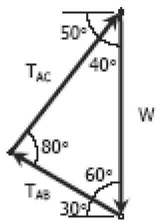
$$\frac{T_{AC}}{\sin 60°} = \frac{W}{\sin 80°}$$

$T_{AC} = 0.8794W$

$T_{AC} = \sigma_{AC}A_{AC}$

$0.8794W = 30(0.5)$

$W = 17.1$ kips



Force polygon of forces on knot A

Safe load $W = 17.1$ kips

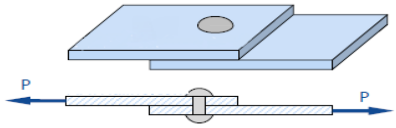
2. Shear stress

This type of stress is developed if the applied force is parallel to the resisting area. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress and it can be calculated by:

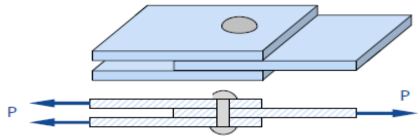
$$\tau = \frac{V}{A}$$

where τ is the shear stress, V is the shearing force which passes parallel to the resisting area and A the area being sheared.

Figure blow shows the shear occur in bolt join two plates which is called single shear and the shear occur in bolt join three plates which is called double shear.



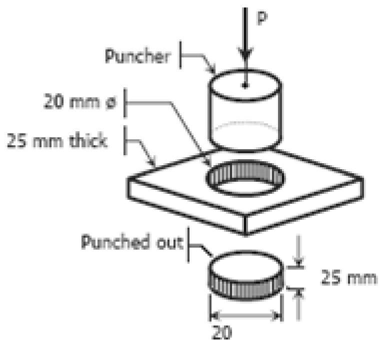
Single Shear



Double Shear

Ex.1. What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m^2.

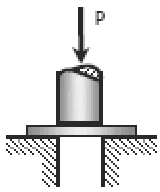
Solution:



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$
$$P = 350[\pi(20)(25)]$$
$$= 549\ 778.7\ \text{N}$$
$$= \mathbf{549.8\ kN}$$

Ex.2:.As in Fig. below, a hole is to be punched out of a plate having a shearing strength of 40 ksi. The compressive stress in the punch is limited to 50 ksi. (a) Compute the maximum thickness of plate in which a hole 2.5 inches in diameter can be punched. (b) If the plate is 0.25 inch thick, determine the diameter of the smallest hole that can be punched.



Solution:

(a) Maximum thickness of plate:

Based on puncher strength:

$$P = \sigma A$$
$$= 50[\tfrac{1}{4}\pi(2.5^2)]$$
$$= 78.125\pi \text{ kips} \rightarrow \text{Equivalent shear force of the plate}$$

Based on shear strength of plate:

$$V = \tau A \qquad \rightarrow V = P$$
$$78.125\pi = 40[\pi(2.5t)]$$
$$t = 0.781 \text{ inch}$$

(b) Diameter of smallest hole:

Based on compression of puncher:

$$P = \sigma A$$
$$= 50(\tfrac{1}{4}\pi d^2)$$
$$= 12.5\pi d^2 \qquad \rightarrow \text{Equivalent shear force for plate}$$

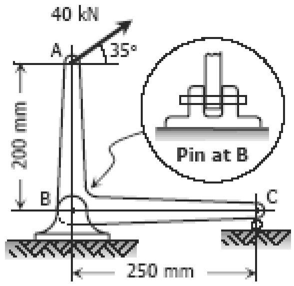
Based on shearing of plate:

$V = \tau A$ → V = P

$12.5\pi d^2 = 40[\pi d(0.25)]$

$d = \mathbf{0.8\ in}$

Ex.3:Compute the shearing stress in the pin at B for the member supported as shown in Fig. below. The pin diameter is 20 mm.



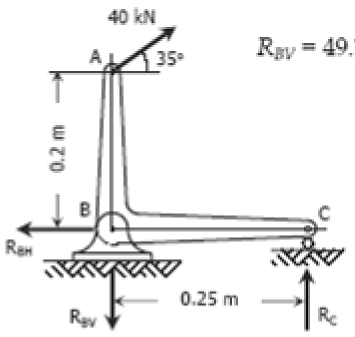
Solution:

From the FBD:

$\Sigma M_C = 0$

$0.25R_{BV} = 0.25(40 \sin 35°) + 0.2(40 \cos 35°)$

$R_{BV} = 49.156$ kN



Free Body Diagram

$\Sigma F_H = 0$

$R_{BH} = 40 \cos 35°$

$= 32.766$ kN

$R_B = \sqrt{R_{BH}{}^2 + R_{BV}{}^2}$

$= \sqrt{32.766^2 + 49.156^2}$

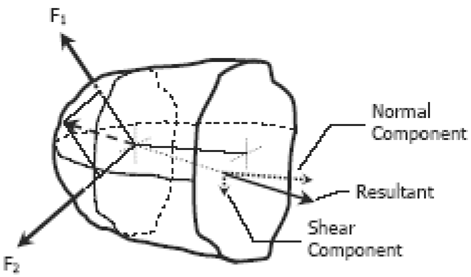
$= 59.076$ kN → shear force of pin at B

$V_B = \tau_B A$ → double shear

$59.076\ (1000) = \tau_B \{2[\frac{1}{4}\pi(20^2)]\}$

$\tau_B = \mathbf{94.02\ MPa}$

Ex.4: Two blocks of wood, width w and thickness t, are glued together along the joint inclined at the angle θ as shown in Fig. Ex.4. Using the free-body diagram concept in Fig. Ex4.a, show that the shearing stress on the glued joint is τ = P sin 2θ/2A, where A is the cross- sectional area.



Normal and shear components of the resultant on arbitrary section.

Fig.Ex.4.A

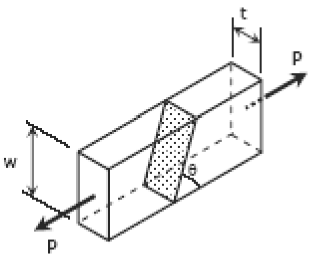
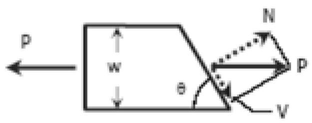


Fig.Ex.4

Solution:



Shear area, $A_{shear} = t\,(w \csc\theta)$

$= tw \csc\theta$

$= A \csc\theta$

Shear force, $V = P\cos\theta$

$V = \tau A_{shear}$

$P\cos\theta = \tau(A\csc\theta)$

$\tau = P\sin\theta\cos\theta\,/\,A$

$= P\,(2\sin\theta\cos\theta)\,/\,2A$

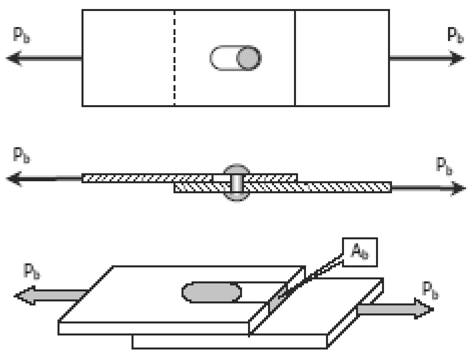
$= P\sin 2\theta\,/\,2A$ (*ok!*)

3. Bearing stress

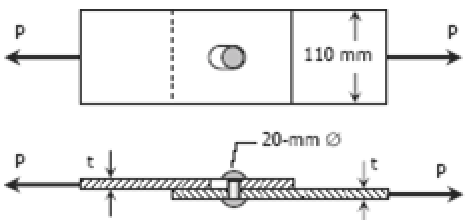
Another type of simple stress is the bearing stress, it is the contact pressure between two separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces.

$$\sigma_b = \frac{P_b}{A_b}$$

where σ_b is the bearing stress, P_b is the bearing force which contact of the area A_b being sheared as shown in the Fig. below:



Ex.1: In Fig. shown below, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine (a) the minimum thickness of each plate; and (b) the largest average tensile stress in the plates.



Solution:

(a) From shearing of rivet:

$P = \tau A_{rivets}$

$= 60[\frac{1}{4}\pi(20^2)]$

$= 6000\pi$ N

From bearing of plate material:

$P = \sigma_b A_b$

$6000\pi = 120(20t)$

t = **7.85 mm**

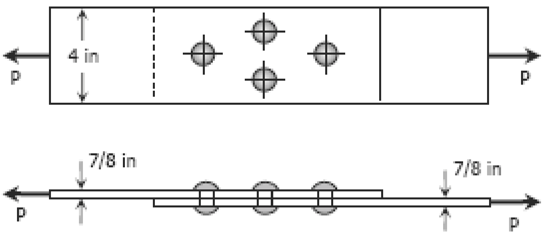
(b) Largest average tensile stress in the plate:

$P = \sigma A$

$6000\pi = \sigma[7.85(110 - 20)]$

σ = **26.67 MPa**

Ex.2 :The lap joint shown in Fig. shown below is fastened by four ¾-in.-diameter rivets. Calculate the maximum safe load P that can be applied if the shearing stress in the rivets is limited to 14 ksi and the bearing stress in the plates is limited to 18 ksi. Assume the applied load is uniformly distributed among the four rivets.



Solution:

Based on shearing of rivets:

$P = \tau A$

$P = 14[4(\frac{1}{4}\pi)(\frac{3}{4})^2]$

$P = 24.74$ kips

Based on bearing of plates:

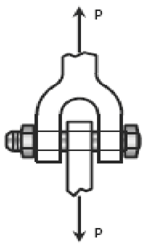
$P = \sigma_b A_b$

$P = 18[4(\frac{3}{4})(\frac{7}{8})]$

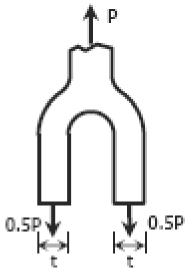
$P = 47.25$ kips

Safe load P = **24.74 kips**

Ex.3:In the clevis shown in Fig. below, find the minimum bolt diameter and the minimum thickness of each yoke that will support a load P = 14 kips without exceeding a shearing stress of 12 ksi and a bearing stress of 20 ksi.



Solution:



For shearing of rivets (double shear)

$P = \tau A$

$14 = 12[2(\frac{1}{4}\pi d^2)]$

$d = \mathbf{0.8618\ in}$ → diameter of bolt

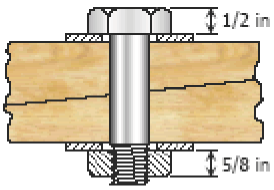
For bearing of yoke:

$P = \sigma_b A_b$

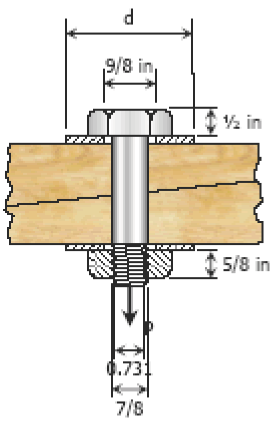
$14 = 20[2(0.8618t)]$

$t = \mathbf{0.4061\ in}$ → thickness of yoke

Ex4.:A 7/8-in.-diameter bolt, having a diameter at the root of the threads of 0.731 in., is used to fasten two timbers together as shown in Fig. shown below. The nut is tightened to cause a tensile stress of 18 ksi in the bolt. Compute the shearing stress in the head of the bolt and in the threads. Also, determine the outside diameter of the washers if their inside diameter is 9/8 in. and the bearing stress is limited to 800 psi.



Solution:



Tensile force on the bolt:

$P = \sigma A = 18[\frac{1}{4}\pi(\frac{7}{8})^2]$

$P = 10.82$ kips

Shearing stress in the head of the bolt:

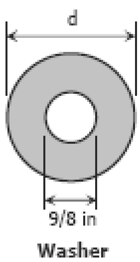
$\tau = \dfrac{P}{A} = \dfrac{10.82}{\pi(\frac{7}{8})(\frac{1}{2})}$

$\tau = 7.872$ ksi

Shearing stress in the threads:

$\tau = \dfrac{P}{A} = \dfrac{10.82}{\pi(0.731)(\frac{5}{8})}$

$\tau = 7.538$ ksi



Outside diameter of washer

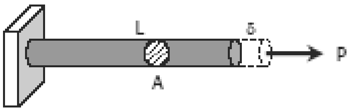
$P = \sigma_b A_b$

$10.82(1000) = 800\{\frac{1}{4}\pi[d^2 - (\frac{9}{8})^2]\}$

$d = \mathbf{4.3\ in}$

Strain
Simple Strain

Strain is the ratio of the change in length caused by the applied force, to the original length, it is also known as unit deformation,



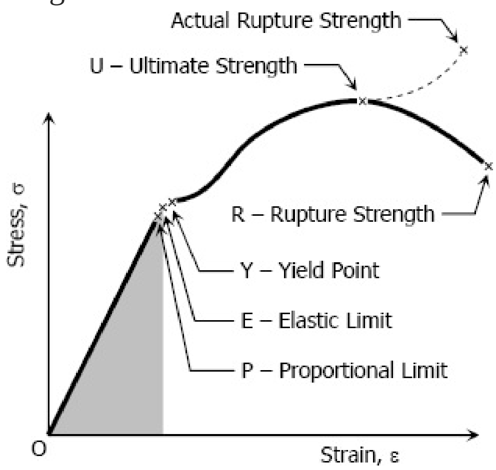
Strain is calculated by : $\varepsilon = \frac{\delta}{L}$

where ε is the strain , δ is the deformation and L is the original length, thus ε is dimensionless.

Stress-Strain Diagram

Suppose that a metal specimen be placed in tension-compression testing machine. As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of the load and this is continued until failure of the specimen takes place. Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain ε can be obtained. The graph of these quantities with the stress σ along the y-axis and the strain ε along the x-axis is called the stress-strain diagram. The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium carbon structural steel.

Metallic engineering materials are classified as either ductile or brittle materials. A ductile material is one having relatively large tensile strains up to the point of rupture like structural steel and aluminum, whereas brittle materials has a relatively small strain up to the point of rupture like cast iron and concrete. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes.



Proportional limit

From the origin O to the point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing was first noticed by Sir Robert Hooke in 1678 and is called Hooke's Law that within the proportional limit, the stress is directly proportional to strain or

$$\sigma \propto \varepsilon \quad \text{or} \quad \sigma = k\varepsilon$$

The constant of proportionality k is called the Modulus of Elasticity E or Young's Modulus and is equal to the slope of the stress-strain diagram from O to P. Then

$$\sigma = E\varepsilon$$

Elastic limit

The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may e developed such that there is no permanent or residual deformation when the load is entirely removed.

Elastic and plastic range

The region in stress-strain diagram from O to P is called the elastic range. The region from P to R is called the plastic range.

Yield point

Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

Ultimate strength

The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.

Rapture strength

Rapture strength is the strength of the material at rupture. This is also known as the breaking strength.

Axial deformation

In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by

$\sigma=E\varepsilon$

since ($\sigma = P / A$) and ($\varepsilon = \delta / L$), then $P / A = E\,\delta / L$. Solving for δ,

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit. If however, the cross- sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.

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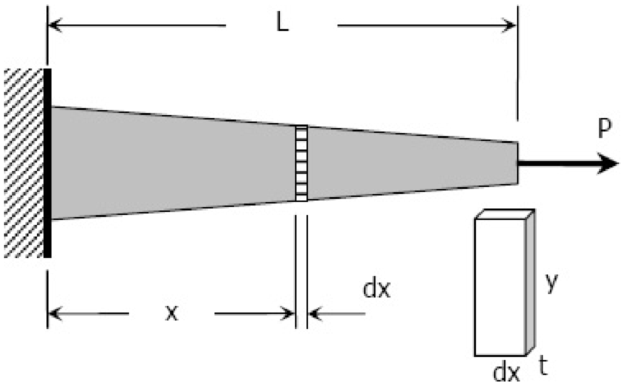


$$\delta = \frac{P}{E} \int_0^L \frac{dx}{L}$$

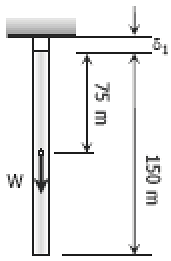
where A = ty

y o t, if variable, must be expressed in terms of x.

Examples:

Ex1: A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and $E = 200 \times 10^3$ MN/m^2, find the total elongation of the rod.

Solution:



Let δ = total elongation

δ_1 = elongation due to its own weight

δ_2 = elongation due to applied load

$\delta = \delta_1 + \delta_2$

$$\delta_1 = \frac{PL}{AE}$$

Where: P = W = 7850(1/1000)3(9.81)[300(150)(1000)]

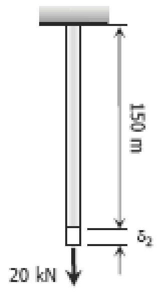
P = 3465.3825 N

L = 75(1000) = 75 000 mm

A = 300 mm^2

E = 200 000 MPa

$$\delta_1 = \frac{3465.3825\ (75000)}{300\ (200\ 000)} = 4.33\ \text{mm}$$



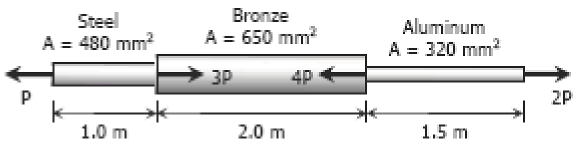
$$\delta_2 = \frac{PL}{AE}$$

Where: P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm²
E = 200 000 MPa

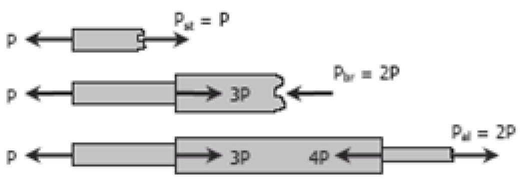
$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50 \text{ mm}$$

Total elongation:
$\delta = 4.33 + 50 = \mathbf{54.33\ mm}$

Ex2: A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use E_{st} = 200 GPa, E_{al} = 70 GPa, and E_{br} = 83 GPa.



Solution:



Based on allowable stresses:

Steel:
$P_{st} = \sigma_{st} A_{st}$
$P = 140(480) = 67\,200$ N
$P = 67.2$ kN

Bronze:
$P_{br} = \sigma_{br} A_{br}$
$2P = 120(650) = 78\,000$
$P = 39\,000 \text{ N} = 39$ kN

Aluminum:
$P_{al} = \sigma_{al} A_{al}$
$2P = 80(320) = 25\,600$ N
$P = 12\,800 \text{ N} = 12.8$ kN

Based on allowable deformation:
(steel and aluminum lengthens, bronze shortens)

$\delta = \delta_{st} - \delta_{br} + \delta_{al}$

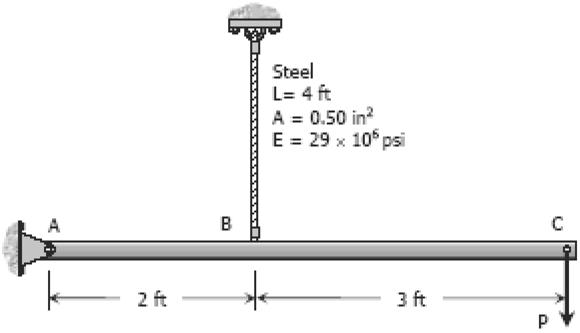
$$3 = \frac{P(1000)}{480(200000)} - \frac{2P(2000)}{650(70000)} + \frac{2P(1500)}{320(83000)}$$

$$3 = \left(\frac{1}{96000} - \frac{1}{11375} + \frac{3}{26560}\right)P$$

$P = 84\,610.99 \text{ N} = 84.61$ kN

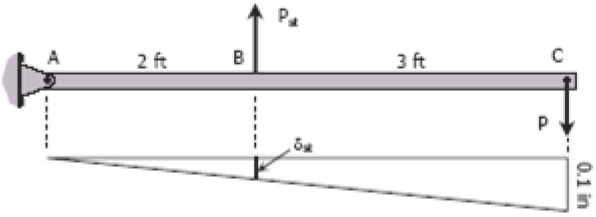
Use the smallest value of P, $P = \mathbf{12.8\ kN}$

Ex3: The rigid bar ABC shown in Fig. below is hinged at A and supported by a steel rod at B. Determine the largest load P that can be applied at C if the stress in the steel rod is limited to 30 ksi and the vertical movement of end C must not exceed 0.10 in.



Solution:

Free body and deformation diagrams:



Based on maximum stress of steel rod:

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

$P = 0.4\sigma_{st}A_{st}$

$P = 0.4[30(0.50)]$

$P = 6$ kips

Based on movement at C:

$$\frac{\delta_{st}}{2} = \frac{0.1}{5}$$

$\delta_{st} = 0.04$ in

$$\frac{P_{st}L}{AE} = 0.04$$

$$\frac{P_{st}(4\times 12)}{0.50(29\times 10^6)} = 0.04$$

$P_{st} = 12\,083.33$ lb

$\Sigma M_A = 0$

$5P = 2P_{st}$

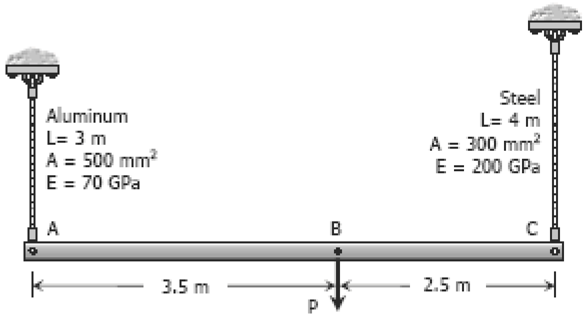
$P = 0.4P_{st}$

$P = 0.4(12\,083.33)$

$P = 4833.33$ lb $= 4.83$ kips

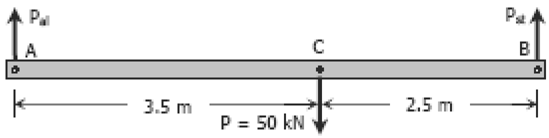
Use the smaller value, P = **4.83 kips**

Ex.3: The rigid bar AB, attached to two vertical rods as shown in Fig. below, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.



Solution:

Free body diagram:



For aluminum:

$[\Sigma M_B = 0]$ $\quad 6P_{al} = 2.5(50)$

$P_{al} = 20.83$ kN

$\left[\delta = \dfrac{PL}{AE}\right]_{al}$ $\quad \delta_{al} = \dfrac{20.83(3)1000^2}{500(70000)}$

$\delta_{al} = 1.78$ mm

For steel:

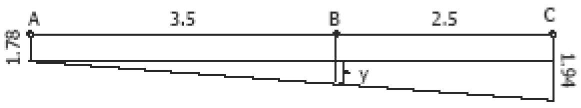
$[\Sigma M_A = 0]$ $\quad 6P_{st} = 3.5(50)$

$P_{st} = 29.17$ kN

$\left[\delta = \dfrac{PL}{AE}\right]_{st}$ $\quad \delta_{st} = \dfrac{29.17(4)1000^2}{300(200000)}$

$\delta_{st} = 1.94$ mm

Movement diagram:



$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$

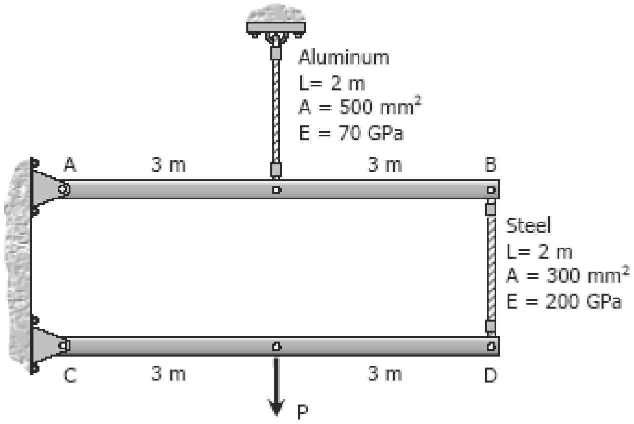
$y = 0.09$ mm

δ_B = vertical movement of P

$\delta_B = 1.78 + y = 1.78 + 0.09$

$\delta_B =$ **1.87 mm**

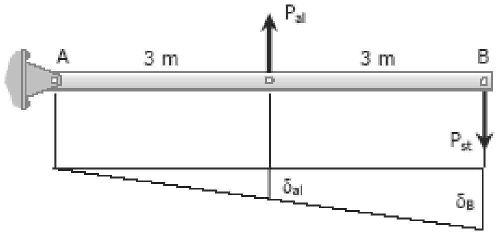
Ex.4: The rigid bars AB and CD shown in Fig. below are supported by pins at A and C and the two rods. Determine the maximum force P that can be applied as shown if its vertical movement is limited to 5 mm. Neglect the weights of all members.



Solution:

$[\Sigma M_A = 0]$ $\quad 3P_{al} = 6P_{st}$

$P_{al} = 2P_{st}$



FBD and movement diagram of bar AB

By ratio and proportion:

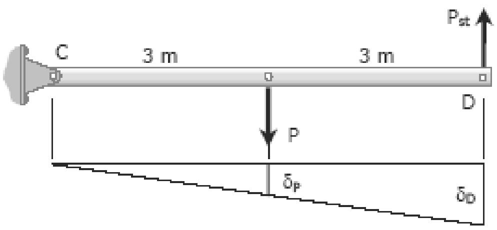
$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$$

$$\delta_B = 2\delta_{al} = 2\left[\frac{PL}{AE}\right]_{al}$$

$$\delta_B = 2\left[\frac{P_{al}(2000)}{500(70\,000)}\right]$$

$$\delta_B = \tfrac{1}{8750}\,P_{al} = \tfrac{1}{8750}\,(2P_{st})$$

$$\delta_B = \tfrac{1}{4375}\,P_{st} \rightarrow \text{movement of B}$$



FBD and movement diagram of bar CD

Movement of D:

$$\delta_D = \delta_{st} + \delta_B = \left[\frac{PL}{AE}\right]_{st} + \tfrac{1}{4375}\,P_{st}$$

$$\delta_D = \frac{P_{st}(2000)}{300(200\,000)} + \tfrac{1}{4375}\,P_{st}$$

$$\delta_D = \tfrac{11}{42\,000}\,P_{st}$$

$[\Sigma M_C = 0]$ $\quad 6P_{st} = 3P$

$P_{st} = \tfrac{1}{2}\,P$

By ratio and proportion:

$$\frac{\delta_P}{3} = \frac{\delta_D}{6}$$

$$\delta_P = \tfrac{1}{2}\,\delta_D = \tfrac{1}{2}\,(\tfrac{11}{42\,000}\,P_{st})$$

$$\delta_P = \tfrac{11}{84\,000}\,P_{st}$$

$$5 = \tfrac{11}{84\,000}\,(\tfrac{1}{2}\,P)$$

$P = 76\,363.64$ N = **76.4 kN**

Ex.5: The following data were recorded during the tensile test of a 14-mm-diameter mild steel rod. The gage length was 50 mm.

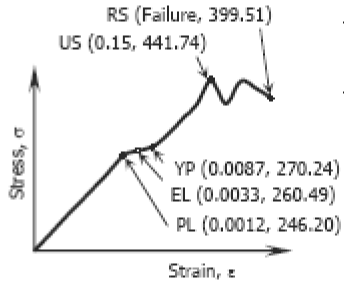
Load (N)	Elongation (mm)	Load (N)	Elongation (mm)
0	0	46200	1.25
6310	0.010	52400	2.50
12600	0.020	58500	4.50
18800	0.030	68000	7.50
25100	0.040	59000	12.50
31300	0.050	67800	15.50
37900	0.060	65000	20.00
40100	0.163	61500	Fracture
41600	0.433		

Plot the stress-strain diagram and determine the following mechanical properties: (a) proportional limits; (b) modulus of elasticity; (c) yield point; (d) ultimate strength; and(e) rupture strength.

Solution:

Area, $A = \frac{1}{4}\pi(14)^2 = 49\pi\ mm^2$; Length, $L = 50$ mm

Strain = Elongation/Length; Stress = Load/Area



Stress-Strain Diagram
(not drawn to scale)

PL = Proportional Limit
EL = Elastic Limit
YP = Yield Point
US = Ultimate Strength
RS = Rupture Strength

Load (N)	Elongation (mm)	Strain (mm/mm)	Stress (MPa)
0	0	0	0
6310	0.010	0.0002	40.99
12600	0.020	0.0004	81.85
18800	0.030	0.0006	122.13
25100	0.040	0.0008	163.05
31300	0.050	0.001	203.33
37900	0.060	0.0012	246.20
40100	0.163	0.0033	260.49
41600	0.433	0.0087	270.24
46200	1.250	0.025	300.12
52400	2.500	0.05	340.40
58500	4.500	0.09	380.02
68000	7.500	0.15	441.74
59000	12.500	0.25	383.27
67800	15.500	0.31	440.44
65000	20.000	0.4	422.25
61500	Failure		399.51

From stress-strain diagram:

(a) Proportional Limit = **246.20 MPa**

(b) Modulus of Elasticity

E = slope of stress-strain diagram within proportional limit

$$E = \frac{246.20}{0.0012} = 205\,166.67 \text{ MPa}$$

$E = 205.2$ GPa

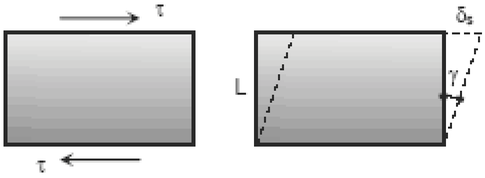
(c) Yield Point = **270.24 MPa**

(d) Ultimate Strength = **441.74 MPa**

(e) Rupture Strength = **399.51 MPa**

Shearing deformation

Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape.



The change in angle at the corner of an original rectangular element is called the **shear strain** and is expressed as

$$\gamma = \frac{\delta_s}{L}$$

The ratio of the shear stress τ and the shear strain γ is called the **modulus of elasticity** in shear or modulus of rigidity and is denoted as G, in MPa.

$$G = \frac{\tau}{\gamma}$$

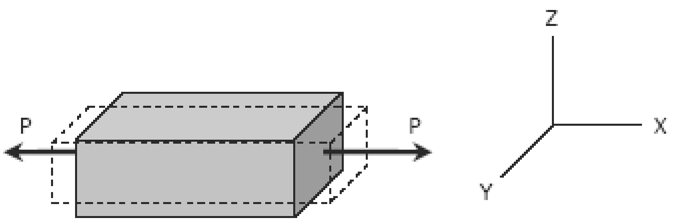
The relationship between the shearing deformation and the applied shearing force is

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

where V is the shearing force acting over an area A_S.

Poisson' s ratio

When a bar is subjected to a tensile loading there is an increase in length of the bar in the direction of the applied load, but there is also a decrease in a lateral dimension perpendicular to the load. The ratio of the sidewise deformation (or strain) to the longitudinal deformation (or strain) is called the Poisson's ratio and is denoted by ν. For most steel, it lies in the range of 0.25 to 0.3, and 0.20 for concrete.



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

where ε_x is strain in the x-direction and ε_y and ε_z are the strains in the perpendicular direction. The negative sign indicates a decrease in the transverse dimension when εx is positive.

Biaxial deformation

If an element is subjected simultaneously by ensile stresses, σx and σy, in the x and y directions, the strain in the x-direction is σx / E and the strain in the y direction is σy / E. Simultaneously, the stress in the y direction will produce a lateral contraction on the x direction of the amount -ν εy or -ν σy/E. The resulting strain in the x direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \quad \text{or} \quad \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1-\nu^2}$$

and

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \quad \text{or} \quad \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1-\nu^2}$$

Triaxial deformation

If an element is subjected simultaneously by three mutually perpendicular normal stresses σx, σy, and σz, which are accompanied by strains εx, εy, and εz, respectively,

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Tensile stresses and elongation are taken as positive. Compressive stresses and contraction are taken as negative.

Relationship Between E, G, and ν

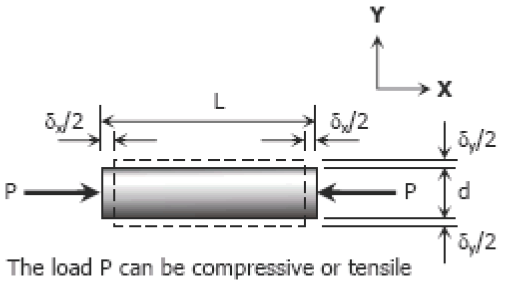
The relationship between modulus of elasticity E, shear modulus G and Poisson's ratio ν

is:

$$G = \frac{E}{2(1+\nu)}$$

Ex.1: A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is 4Pν / πEd.

Solution:



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\varepsilon_x$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

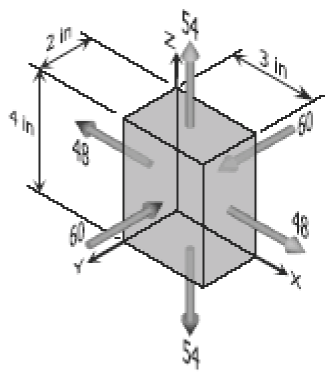
$$\frac{\delta_y}{d} = -\nu\frac{-P}{AE}$$

$$\delta_y = \nu\frac{Pd}{\frac{1}{4}\pi d^2 E}$$

$$\delta_y = \frac{4P\nu}{\pi E d} \qquad ok!$$

Ex.2: A rectangular steel block is 3 inches long in the x direction, 2 inches long in the y direction, and 4 inches long in the z direction. The block is subjected to a triaxial loading of three uniformly distributed forces as follows: 48 kips tension in the x direction, 60 kips compression in the y direction, and 54 kips tension in the z direction. If ν = 0.30 and E = 29×10^6 psi, determine the single uniformly distributed load in the x direction that would produce the same deformation in the y direction as the original loading.

Solution:



For triaxial deformation (tensile triaxial stresses):
(compressive stresses are negative stresses)

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\sigma_x = \frac{P_x}{A_{yz}} = \frac{48}{4(2)} = 6.0 \text{ ksi (tension)}$$

$$\sigma_y = \frac{P_y}{A_{xz}} = \frac{60}{4(3)} = 5.0 \text{ ksi (compression)}$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{54}{2(3)} = 9.0 \text{ ksi (tension)}$$

$$\varepsilon_y = \frac{1}{29\times10^6}[-5000 - 0.30(6000 + 9000)]$$

$$\varepsilon_y = -3.276 \times 10^{-4}$$

ε_y is negative, thus tensile force is required in the *x*-direction to produce the same deformation in the *y*-direction as the original forces.

For equivalent single force in the *x*-direction:
(uniaxial stress)

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$-\nu\varepsilon_x = \varepsilon_y$$

$$-\nu\frac{\sigma_x}{E} = \varepsilon_y$$

$$-0.30\left(\frac{\sigma_x}{29\times10^6}\right) = -3.276\times10^{-4}$$

$$\sigma_x = 31\ 666.67\text{ psi}$$

$$\sigma_x = \frac{P_x}{4(2)} = 31\ 666.67$$

$$P_x = 253\ 333.33\text{ lb (tension)}$$

$$P_x = \mathbf{253.33\ kips\ (tension)}$$

Ex.3:For the block loaded triaxially as described in next example, find the uniformly distributed load that must be added in the x direction to produce no deformation in the z direction .

Solution:

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$\sigma_x = 6.0$ ksi (tension)

$\sigma_y = 5.0$ ksi (compression)

$\sigma_z = 9.0$ ksi (tension)

$$\varepsilon_z = \frac{1}{29\times10^6}[9000 - 0.3(6000 - 5000)]$$

$$\varepsilon_z = 2.07\times10^{-5}$$

ε_z is positive, thus positive stress is needed in the x-direction to eliminate deformation in z-direction.

The application of loads is still simultaneous:
(No deformation means zero strain)

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] = 0$$

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$\sigma_y = 5.0$ ksi → (compression)

$\sigma_z = 9.0$ ksi → (tension)

$$9000 = 0.30(\sigma_x - 5000)$$

$$\sigma_x = 35\ 000\text{ psi}$$

$$\sigma_{added} + 6000 = 35\ 000$$

$$\sigma_{added} = 29\ 000\text{ psi}$$

$$\frac{P_{added}}{2(4)} = 29\ 000$$

$$P_{added} = 232\ 000\text{ lb}$$

$$P_{added} = \mathbf{232\ kips}$$

Ex.4. A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

Solution:

σ_y = longitudinal stress

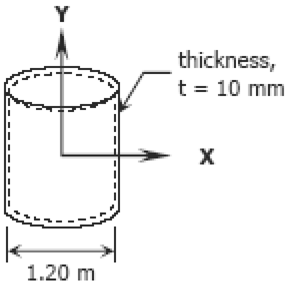
$$\sigma_y = \frac{pD}{4t} = \frac{1.5(1200)}{4(10)}$$

$\sigma_y = 45$ MPa

σ_x = tangential stress

$$\sigma_x = \frac{pD}{2t} = \frac{1.5(1200)}{2(10)}$$

$\sigma_x = 90$ MPa



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_x = \frac{90}{200000} - 0.3\left(\frac{45}{200000}\right)$$

$\varepsilon_x = 3.825 \times 10^{-4}$

$$\varepsilon_x = \frac{\Delta D}{D}$$

$\Delta D = \varepsilon_x D = (3.825 \times 10^{-4})(1200)$

$\Delta D = \mathbf{0.459\ mm}$